



Influence of inner diameter and position of phase adjuster on the performance of the thermo-acoustic Stirling engine



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HIGHLIGHTS

- The effects of inner diameter and position of PA on the engine were investigated systematically.
- PA with suitable position and inner diameter can increase pressure amplitude and acoustic power of the engine.
- Pressure drop generated by PA and the variation of sound direct current in the loop were also analyzed.
- The higher acoustic power is accompanied by the bigger sound direct current.

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ABSTRACT

A thermo-acoustic Stirling engine can convert thermal energy into useful acoustic power without moving parts, which can be used as driving power for the refrigerators and the generators. However, the low efficiency of the thermo-acoustic engine limits its commercial application. In this paper, to promote the performance of the thermo-acoustic engine, a phase adjuster (PA) is introduced by narrowing the cross-sectional area on the basis of the previous studies. The influences of the position and inner diameter of PA on the resonant frequency, pressure amplitude, acoustic power, sound direct current (DC) and efficiency are studied systematically. It is found that the maximal pressure amplitude, acoustic power and efficiency with optimal PA could be increased by 3%, 4.5% and 10% respectively compared to those without PA. In addition, the effect of PA on DC flow is also discussed in detail. Results show that the sound DC decreases with the farther position and smaller inner diameter of PA. Therefore, the suitable inner diameter and position can improve the efficiency of the engine, which needs to be weighed when designing or optimizing PA.

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1. Introduction

A thermo-acoustic engine based on the thermo-acoustic effect can realize the heat-to-sound conversion without moving parts [1]. In devices of this type, an acoustic wave interacts with internal solid structures such as regenerators and stacks to create acoustic power due to an imposed temperature gradient. In addition, it can operate with working fluids like noble gases that are benign to the environment. The low-grade energy such as the solar energy and the industrial waste heat can be used as thermal sources to produce the temperature gradient combined with an ambient heat exchanger. Therefore, developing the thermo-acoustic technology

is meaningful to mitigate the energy crisis and to keep the ecological balance.

In a thermo-acoustic engine, the working fluid oscillates at a fixed frequency when the temperature gradient in the regenerator (stacks) reaches a critical value. In the early study of the thermo-acoustic engine, the structural size was very large, the onset temperature was high and the sound intensity density was usually low. As the new strategies and novel structures, such as pressure disturbance, multi-stage traveling wave thermo-acoustic engine and so on, were proposed, the thermo-acoustic engine has developed toward the miniaturization, low onset temperature difference and high sound intensity. In a viewpoint of saving energy, the onset temperature difference should be as low as possible. Therefore, improving the efficiency of heat-to-sound energy conversion has become the subject of much research. To achieve this goal, studies on the thermo-acoustic effect, unsteady heat transfer in the porous

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media, and acoustic impedance matching have been carried out by numerical and experimental methods [2–6].

In a thermo-acoustic system, the heat-to-sound conversion is controlled by two parameters: $\omega\tau$ (as defined in Eq. (1)), and the phase angle between sound pressure and velocity [7].

$$\omega\tau = \left(\frac{r}{\delta_\alpha}\right)^2 \quad (1)$$

where ω denotes angular frequency, τ is the thermal relaxation time, r is the hydraulic radius of the regenerator, and δ_α is the thermal penetration thickness. For the standing wave thermo-acoustic engine, the stack plate space is about 2–4 times in thickness of the thermal penetration to induce a significant time delay between the gas motion and the gas thermal expansion (or contraction) to meet the Rayleigh's criterion. For the traveling wave thermo-acoustic engine, the hydraulic radius is much smaller than the thermal penetration thickness to ensure a near isothermal condition in the regenerator.

The phase angle is the other parameter to determine the efficiency of heat-to-sound energy conversion [8,9]. Ceperley found that the efficiency of heat-to-sound energy conversion was higher when the phase angle between pressure and particle velocity was around zero [10]. Yazaki succeeded in realizing this thermo-acoustic setup by using the looped tube in 1998 [11]. Biwa studied the thermo-acoustic phenomena in detail by combining experimental methods with computing methods [12]. He identified that the efficiency would be higher if the phase angle between pressure and particle velocity was closer to 0. In consideration of the importance of phase angle, a number of strategies had been proposed to adjust the phase angle. Historically, the first study on the phase angle adjustment could date back to Swift [13]. In his research about the thermo-acoustic Stirling engine, the jet pump was applied to reduce the Gedeon streaming in the loop and achieved a great success [14]. Nevertheless, the sudden change of the cross-sectional area brought by the jet pump could cause serious energy dissipation. Although the adjustment for phase angle was not mentioned explicitly, the jet pump acted as the rudiment of phase adjuster. Barton et al. analyzed power dissipation and time-average pressure through the jet pumps with different geometries [15]. Petculescu et al. investigated the nonlinear effect and minor loss of the jet pump and discussed the influence of different cone half-angles [16]. In recent years, Sakamoto had done much research on phase angle adjustment methods and proposed several feasible approaches for phase angle adjustment [17,18]. For example, one approach was to apply a diverging tube in the looped thermo-acoustic engine to adjust the phase angle. The temperature difference of both ends in the stack reduced to 345 °C from 845 °C [19]. Another was to introduce a length of circular tube with sudden contraction of cross-section which is called phase adjuster (PA) in

the thermo-acoustic cooling system. PA adjusted the phase difference by narrowing the cross-sectional area, thus improving the cooling effect. Subsequently, they investigated the influence of inner diameter of PA on heat-to-sound energy conversion efficiency in a Loop-Tube-Type thermo-acoustic system [19]. Sahashi et al. studied further the working mechanism of PA in a loop-tube-type thermo-acoustic cooling system by experiments [20]. However, it was challenging to control the phase angle between pressure and particle velocity because of the self-excitation oscillation of the working fluid. To date, the working mechanism of PA was not completely clear. To the best of our knowledge, studies on the thermo-acoustic Stirling engine with PA were not reported in the literature.

In this paper, inspired by the jet pump and phase adjuster, we introduce a PA in the thermo-acoustic Stirling engine by narrowing the cross-sectional area. The abrupt change of the cross-sectional area with PA is not larger than that with the jet pump, resulting in fewer energy losses than those generated by the jet pump. Firstly, the phase adjustment of the PA is verified. Secondly, the influences of the position and inner diameter of PA on the resonance frequency, pressure amplitude, acoustic power and the efficiency of the TASHE are studied systematically. The impact of PA on sound DC is then analyzed. Finally, the formation of vortexes is also discussed through the streamlines pattern to further understand and help the optimal design of PA structure.

2. Numerical model

In this section, the CFD model for the thermo-acoustic Stirling engine with a PA is described. The governing equations, initial and boundary conditions and numerical algorithm analysis of numerical model are also presented.

2.1. CFD model of the thermo-acoustic Stirling engine

Fig. 1(a) illustrates a thermo-acoustic Stirling engine with a PA. The engine is comprised of a loop and a resonator, which has the advantages of both standing wave and traveling wave. The loop includes a HHX, a regenerator, a TTB and two ambient heat exchangers. PA is placed on the top of the MAHX by narrowing the cross-sectional area. Inspired by the work of Lycklama [21], an axisymmetric grid was applied for the modeling the system to save the computational time. The topological structure of the thermo-acoustic Stirling engine is performed in Fig. 1(b). The original loop is composed of two concentric tubes. The dimensions of the main parts are given in Table 1.

Fig. 2 presents the three-dimensional schematic illustration of PA. L is the length of PA, and d is the diameter of PA which is smaller than that of the tube. Similar to the jet pump [14], the working gas will make a transition from a large area to a small one when flowing

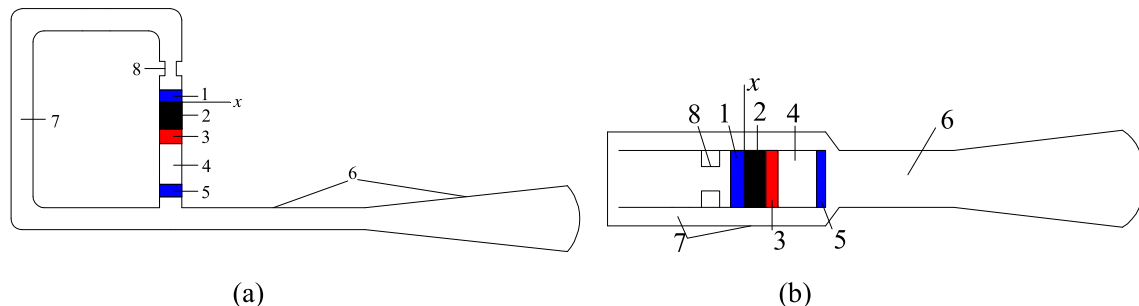


Fig. 1. Schematic illustration of thermo-acoustic Stirling engine (a), and its two dimensional topological structure (b). 1: main ambient heat exchanger (MAHX); 2: regenerator; 3: hot heat exchanger (HHX); 4: thermal buffer tube (TTB); 5: secondary ambient heat exchanger (SAHX); 6: resonator; 7: feedback tube; 8: PA.

Table 1
Dimensions and mesh type of main parts in the thermo-acoustic Stirling engine.

Component	Diameter (m)	Length (m)	Mesh type	Mesh size (mm)
Feedback tube	0.080	2.980	Quad Pave	2
MAHX	0.080	0.045	Quad Map	1
Regenerator	0.080	0.080	Quad Map	0.6
HXX	0.080	0.058	Quad Map	1.2
TBT	0.080	0.240	Quad Pave	2
SAHX	0.080	0.022	Quad Map	1
Resonator	0.080–0.200	5.000	Quad Pave	4

through PA. On one hand, this transition creates a strong flow condition where velocity is accelerated, resulting in the variation of pressure and the phase between pressure and velocity. On the other hand, the transition will bring about an additional pressure drop and dissipation, called “minor losses”. To analyze the influence of PA on the system performance systematically, different positions and inner diameters of PA are investigated and compared. All positions of PA are given relative to the ambient end of the regenerator (marked by position of x in Fig. 1). The main dimensions of PA are shown in Table 2.

2.2. Governing equations

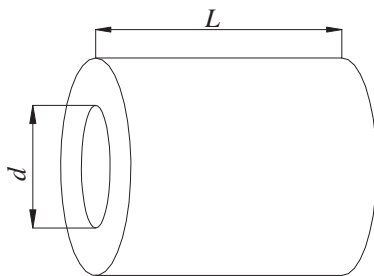
In a traveling wave thermo-acoustic engine, the hydraulic diameter in the regenerator should be much smaller than the thermal penetration for ensuring a good thermal contact between the fluid and solid. Porous zone in FLUENT satisfies this condition and is applied in the regenerator and heat exchangers successfully. The regenerator is assumed to be rigid because the pressure oscillation only occupies 0.03 of the mean pressure and will not cause the deformation of the regenerator. When the working fluid flows through porous zone, pressure drop can be considered by adding the additional momentum loss into the momentum equation, which can avoid the difficulty in modeling complicated physical structure. The additional momentum loss can be expressed as [22]:

$$\vec{S}_i = -\left(\frac{\mu}{\beta} \vec{v} + \frac{1}{2} C_2 \rho_f |\vec{v}| \vec{v}\right) \quad (2)$$

where S_i represents the additional momentum loss in the porous zone; β , C_2 and μ denote the penetration coefficient, inertial resistance coefficient and dynamic viscous, respectively. Some related coefficients applied in the porous zone are given in Table 3.

With Eq. (2), the mass, momentum and energy conservation equations of the working fluid in the porous zone are given by Ref. [22]:

$$\frac{\partial(\epsilon \rho_f)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(\epsilon r \rho_f v_f) + \frac{\partial}{\partial x}(\epsilon \rho_f v_x) = 0 \quad (3)$$

**Fig. 2.** The schematic illustration of PA.**Table 2**
Dimensions and position of PA.

Position – away from the ambient end of the regenerator (m)	The ratio of inner diameter of PA to the diameter of the tube (d/D)					Length (m)
0.43	0.375	0.5	0.625	0.75	0.875	0.045
0.52	0.375	0.5	0.625	0.75	0.875	0.045
0.62	0.375	0.5	0.625	0.75	0.875	0.045

$$\frac{\partial}{\partial t}(\epsilon \rho_f \vec{v}) + \nabla \cdot (\epsilon \rho_f \vec{v} \vec{v}) = -\epsilon \nabla p + \nabla \cdot (\epsilon \tau) - \left(\frac{\mu}{\beta} \vec{v} + \frac{1}{2} C_2 \rho_f |\vec{v}| \vec{v}\right) \quad (4)$$

$$\begin{aligned} \frac{\partial}{\partial t}(\epsilon \rho_f E_f + (1 - \epsilon) \rho_s E_s) + \nabla \cdot (\vec{v}(\rho_f E_f + p)) \\ = \nabla \cdot [(\epsilon k_f + (1 - \epsilon) k_s) \nabla T + (\epsilon \tau \cdot \vec{v})] \end{aligned} \quad (5)$$

where ρ , v , p and T are density, velocity, pressure and temperature, respectively; τ is stress tensor; E and h are internal energy and enthalpy; k and ϵ denote the thermal-conductivity coefficient and the porosity of the porous zone. r and x represent the radial and axis direction. The superscript $\rightarrow$ denotes the vector, the subscript f and s denote the working fluid and the solid material of the porous zone.

For other components, the mass, momentum and energy conservation equations solved by FLUENT are as follows [22]:

$$\frac{\partial(\rho_f)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r}(r \rho_f v_f) + \frac{\partial}{\partial x}(\rho_f v_x) = 0 \quad (6)$$

$$\frac{\partial}{\partial t}(\rho_f \vec{v}) + \nabla \cdot (\rho_f \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\tau) \quad (7)$$

$$\frac{\partial}{\partial t}(\rho_f E) + \nabla \cdot (\vec{v}(\rho_f E + p)) = \nabla \cdot [k \nabla T + (\tau \cdot \vec{v})] \quad (8)$$

$$E = h - \frac{p}{\rho} + \frac{v^2}{2} \quad (9)$$

Eqs. (2)–(9) will be combined with the gas state equation describing the total thermo-acoustic phenomena in a thermo-acoustic Stirling engine.

2.3. Initial and boundary conditions

In the numerical model, there are mainly two conditions to establish the temperature gradient: the constant heat flux and the constant temperature. The condition of constant heat flux means that the constant heat flux is input in the hot heat exchanger. Under this condition, the temperature gradient in the regenerator goes up gradually. Another condition of constant temperature means that the hot and ambient heat exchangers are set at constant temperature, and temperature gradient in the regenerator is decided by

Table 3
Porous materials and coefficients in the porous zone.

Parts	Material	Porosity rate ϵ	β (m^{-2})	C_2 (m^{-1})
Regenerator	Stainless steel	0.5	1.89E-10	4.4592E4
HXX	Copper	0.675	2.16E-7	1.0627E3
Ambient heat exchanger (AHX)	Copper	0.5	2.16E-7	1.0627E3

the thermodynamic process between the gas and the regenerator's material. In this paper, to conveniently compare the performance of the TASHE with/without PA and analyze the influence of PA, the structural parameter and the temperature of the hot and ambient heat exchanger are set to be unchangeable. In order to save computing time, the initial condition of linear temperature gradient in the regenerator is given originally in Table 4. Finally, the stable temperature distribution in the regenerator is obtained by solving energy equation iteratively.

Helium gas (He) is used as the working gas. Table 4 shows some properties of He and initial conditions used in CFD simulation. For boundary conditions, all walls are assumed to have a fixed temperature $T = 300$ K except for those of the HHX, the regenerator, and the TBT which are set to be adiabatic. In addition, no-slip boundary conditions are imposed at all walls.

2.4. Numerical algorithms analysis

For the compressible and unsteady system in this paper, fixed time step with pressure-based solver and second-order implicit time differencing is used [23]. In our double-precision parallel calculation, Pressure-Implicit with Splitting of Operators (PISO) algorithm and second-order upwind spatial difference are selected to obtain the detailed information regarding the pressure and velocity fields. Different from the steady flow, the velocity in the unsteady flow is metabolic, so the Reynolds number based on the amplitude of velocity is used to judge the flow condition just as the Reynolds number in the steady flow. The Reynolds number based on the amplitude of velocity is defined as [24]:

$$Re = \frac{\rho \langle u \rangle d}{\mu} \quad (10)$$

where ρ is the density of the working gas, d is the diameter of the tube, μ is the viscosity coefficient, $\langle u \rangle$ is the velocity amplitude. After the system is under steady condition, the Reynolds numbers of the elbow and the resonator junction are 9,600 and 14,400 respectively, indicating that the flow has been fully turbulent [24]. Therefore, the standard turbulence $k-\epsilon$ model used for this model is valid for the fully developed turbulent flow [22].

To investigate the grid independence, different models with five kinds of meshes with 12,566, 23,900, 62,964, 103,847 and 128,610 nodes are built and compared. The number of grid with 103,847 is chosen since the results for further refining grid has little effect on the results. Considering the function of different parts and their importance in the thermo-acoustic effect, the different mesh element sizes and types have been used in different parts. The mesh type and size is shown in Table 1. The pressure, velocity and temperature information can be obtained by solving governing equations iteratively, so the selection of convergence criteria is very important. To ensure the calculation accuracy, the absolute convergence criteria for mass, momentum, energy, k and ϵ equation, are $1E-05$, $1E-03$, $1E-06$ and $1E-03$, respectively.

Table 4
The properties of gas and initial conditions used in CFD simulation.

Physical properties	Density	Ideal gas
	Viscosity	Constant
	Thermal conductivity	Constant
Initial conditions	Initial mean pressure	1.5 MPa
	Initial velocity	0
	Temperature of the HHX	800 K
	Temperature gradient in the regenerator	$300 + 6250 \times (x - 1.073)$
	temperature of the AHX	300 K

3. Pressure drop and phase modulation of phase adjuster

3.1. Pressure drop

To calculate the time-averaged pressure drop through PA, the local resistance loss coefficients K_B and K_S in the alternating current are assumed to be the same as that in the steady flow. This assumption was verified by Petculescu [16]. Although flow direction varies repeatedly in one cycle, the time of forward and backward direction is equal. Therefore, we can divide one cycle into two half cycles and neglect the variations in density, which is presented in Fig. 3.

According to the dynamic Bernoulli equations, during one half of the cycle shown in Fig. 3(a) [15],

$$p_b + \frac{1}{2} \rho u_b^2 - \frac{1}{2} K_S \rho u_a^2 + \rho \int_b^a \frac{du}{dt} dx = p_a + \frac{1}{2} \rho u_a^2 \quad (11)$$

$$p_a + \frac{1}{2} \rho u_a^2 - \frac{1}{2} K_B \rho u_a^2 = p_c + \rho \int_a^c \frac{du}{dt} dx + \frac{1}{2} \rho u_c^2 \quad (12)$$

During the other half of the cycle as shown in Fig. 3(b), the flow direction changes reversely. The dynamic Bernoulli equations are given by:

$$p_c + \frac{1}{2} \rho u_c^2 - \frac{1}{2} K_S \rho u_a^2 + \rho \int_c^a \frac{du}{dt} dx = p_a + \frac{1}{2} \rho u_a^2 \quad (13)$$

$$p_a + \frac{1}{2} \rho u_a^2 - \frac{1}{2} K_B \rho u_a^2 = p_b + \rho \int_a^b \frac{du}{dt} dx + \frac{1}{2} \rho u_b^2 \quad (14)$$

where p_a , p_b , p_c are pressure at the position of a , b , c ; u_a , u_b , u_c are velocity at the position of a , b , c . K_B and K_S donate the local resistance loss coefficients.

Combining Eqs. (11)–(14), the pressure drop can be given as following:

$$\Delta p = \left[\frac{\rho u_{a,\max}^2}{2} (K_B - K_S - 2) + \frac{\rho}{2} (u_{b,\max}^2 + u_{c,\max}^2) \right] \alpha \quad (15)$$

where

$$\alpha = \frac{1}{Tu_{\max}^2} \int_0^T u^2 dt \quad (16)$$

If the flow is sinusoidal, then $\alpha = 1/2$ and Eq. (15) will become

$$\Delta p = \frac{\rho u_{a,\max}^2}{4} (K_B - K_S - 2) + \frac{\rho}{4} (u_{b,\max}^2 + u_{c,\max}^2) \quad (17)$$

where $u_{a,\max}$, $u_{b,\max}$ and $u_{c,\max}$ are the maximum velocity at the position of a , b , c .

3.2. Phase modulation

Sakamoto et al. [17] proposed diverging tube in a Loop-Tube-Type thermo-acoustic system to control the phase between pressure and velocity by moving the anti-node of pressure. Similar to

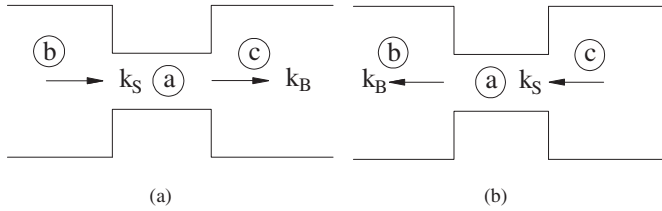


Fig. 3. Alternating current flows through PA periodically. One cycle is divided into two half cycles according to the flow direction, respectively shown in (a) and (b).

the diverging tube, the introduction of PA increases the disturbance by narrowing the cross-sectional area, resulting in the change of phase between pressure and velocity. The pressure node with and without PA is shown in Fig. 4. The intersections of two solid lines and two dotted lines denote the pressure node with and without PA, respectively. It can be seen that the position of the pressure node with PA moves relative to that of the case without PA, which indicates the phase between pressure and velocity is modulated.

4. Results

4.1. Resonant frequency

Fig. 5 shows the variation of the resonant frequency with the position and inner diameter of PA. The dotted line represents the resonant frequency 56.5 Hz without PA. It can be seen that the frequency change after using PA is small, less than 0.3 Hz. According to the formula of the resonant frequency $f = a/\lambda$, the frequency is a function of sound velocity a and wavelength λ . Sound velocity associates closely with the type of gas and the system temperature. λ is determined by the dimension of the resonator. Overall, the introduction of PA has little impact on resonant frequency because the structural change after using PA is so small that it does not arouse the larger change of the resonant frequency.

4.2. Pressure amplitude

The variation of pressure amplitude with the position and inner diameter of PA at the pressure anti-node is presented in Fig. 6. The dotted line represents the pressure amplitude without PA. For a fixed position of PA except for 0.62 m, the pressure amplitude increases initially and then decreases slightly. It can be seen that the pressure amplitude achieves its maximum value when d/D is 0.75. When PA is 0.62 m away from the hot end of the regenerator,

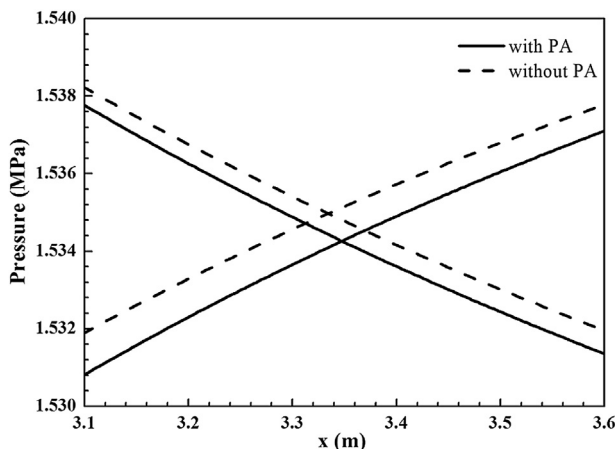


Fig. 4. The pressure node with and without PA.

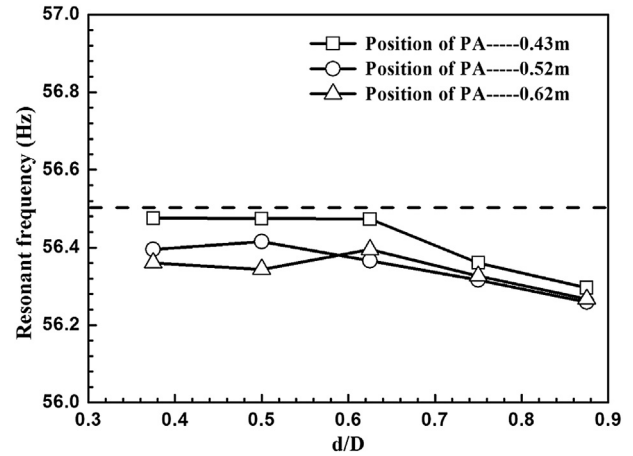


Fig. 5. The variation of resonant frequency with the ratio d/D at the different position of PA.

pressure amplitude increases monotonously with inner diameter of PA, but the increase becomes flat gradually. For a fixed inner diameter of PA, pressure amplitude decreases with an increase in the position of PA except PA with $d/D = 0.875$. Nevertheless, this effect weakens as inner diameter of PA increases. It is observed that the difference of pressure amplitude at three positions of PA is small when the d/D is 0.875. Overall, the maximal pressure amplitude increases by 3% compared to the case without PA. However, pressure amplitude with the $d/D = 0.375$ is smaller than that without PA regardless of the position of PA.

4.3. Acoustic power

Acoustic power characterizes the capacity of heat-to-sound energy conversion at the constant temperature difference between the two ends of the regenerator. The higher the acoustic power is, the stronger the thermo-acoustic effect is. Fig. 7 shows the variation of acoustic power with the position and inner diameter of PA at the cold (a) and hot (b) ends of the regenerator. The variation trend of acoustic power with the position and inner diameter of PA is similar to that of pressure amplitude. Specifically, the acoustic power is higher for the larger inner diameter and the nearer distance from the hot end of the regenerator. When the d/D

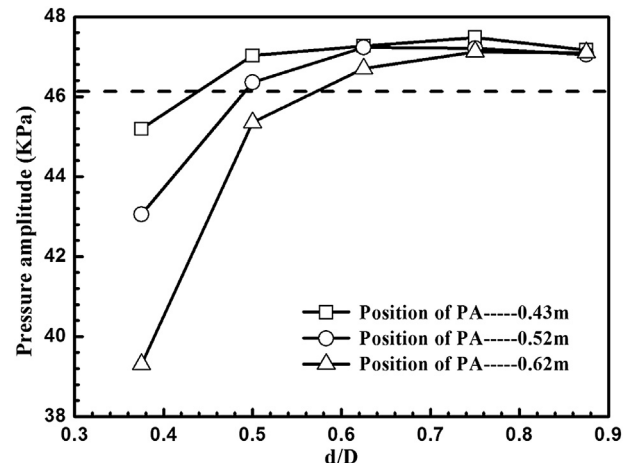


Fig. 6. Pressure amplitude as a function of the ratio d/D at different PA positions.

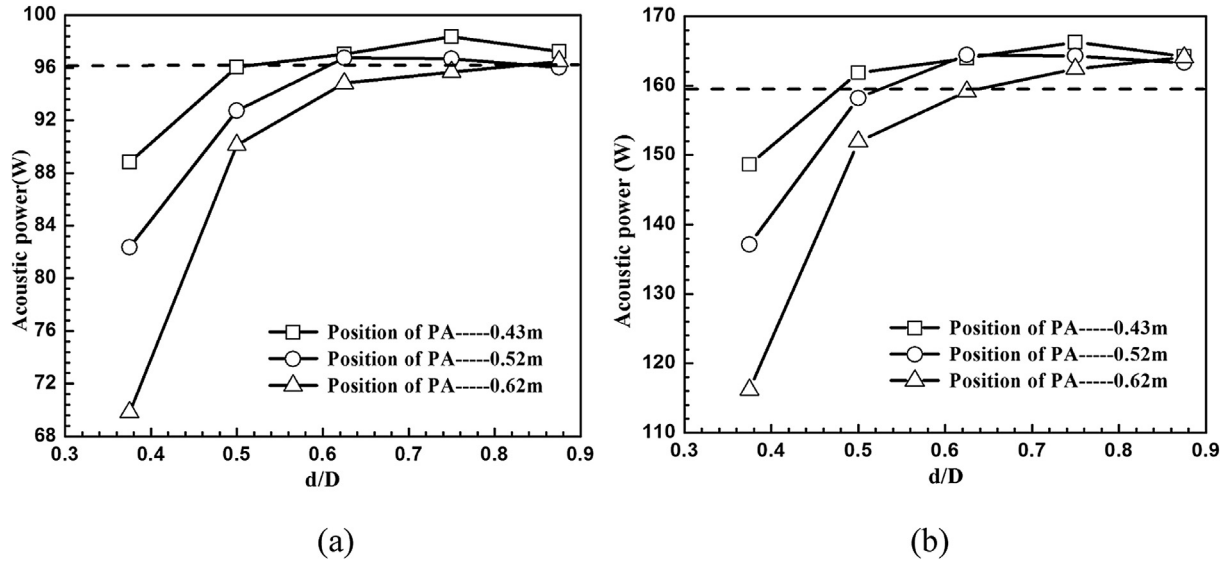


Fig. 7. The variation of acoustic power at the cold end (a) and the hot end (b) of the regenerator with the ratio d/D . The dotted line represents acoustic power at two ends of the regenerator of the engine without PA.

is 0.75, and PA position is 0.43 m from the hot end of the regenerator, the acoustic power achieves its maximum value which increases by 4.5% relative to the case without PA.

4.4. Sound direct current (DC)

The loop in the thermo-acoustic Stirling engine generates “short circuit” for acoustic DC (Gedeon Current) from the hot heat exchanger to the secondary cold heat exchanger. Following Swift [13], the sound DC is given by

$$\dot{M}_2 = \text{Re}[\rho_1 \tilde{U}_1] / 2 + \rho_m U_{20} \quad (18)$$

where $\dot{M}_2$ denotes the sound DC; ρ_1 and ρ_m are the oscillating density and the average density; $\tilde{U}_1$ and U_{20} are the oscillating velocity and the second order time-average volumetric velocity; Re means the real part; the tilde denotes complex conjugation. The first term on the right hand of Eq. (18) is nonzero as long as the system oscillates. If none measures are taken to produce U_{20} to offset sound DC, heat will flow directly from the hot exchanger to the secondary cold heat exchanger, causing unnecessary heat losses.

Fig. 8 illustrates the variation of sound DC with the position and inner diameter of PA. The dotted line represents sound DC without PA and its value is 0.039 kg s^{-1} . The sound DC decreases with the increase in position of PA at a fixed inner diameter of PA. However, the influence of position of PA becomes weak as the inner diameter of PA increases. At a fixed position of PA, the sound DC decreases with inner diameter of PA. When the PA position is 0.63 m away from the hot end of the regenerator, the sound DC with $d/D = 0.375$ decreases by 30% relative to the case without PA. Nevertheless, the sound DC is 0.040 kg s^{-1} higher than that without PA when the d/D is 0.75 or 0.875. Comparing Figs. 7 and 8, it can be found that the sound DC increases while acoustic power increases.

When the alternating current flows through PA, additional pressure drop is generated from a large cross-sectional area to a small one (or the other way around). We use Eq. (17) to calculate the pressure drop, and the variation of pressure drop as inner diameter and position of PA is shown in Fig. 9.

It can be clearly seen that the pressure drop is negative, implying that the U_{20} brought by pressure drop can offset a part of acoustic DC. The absolute value of pressure drop increases with a decrease in inner diameter of PA, and increases with position of PA. According to Eq. (17), pressure drop is determined by the square of particle velocity and increases with particle velocity. On one hand, the particle velocity increases as inner diameter of PA decreases. On the other hand, particle velocity increases with the distance from the hot end of the regenerator. Obviously, the higher negative pressure drop is more helpful to suppress the acoustic DC. Therefore, the higher pressure drop brings about the smaller sound DC comparing Figs. 8 and 9.

4.5. Efficiency

The variation of the efficiency of the engine with inner diameter and position of PA is shown in Fig. 10. It can be seen that the efficiency firstly increases and then decreases with the increase in inner diameter of PA, and decreases with the increase in position of

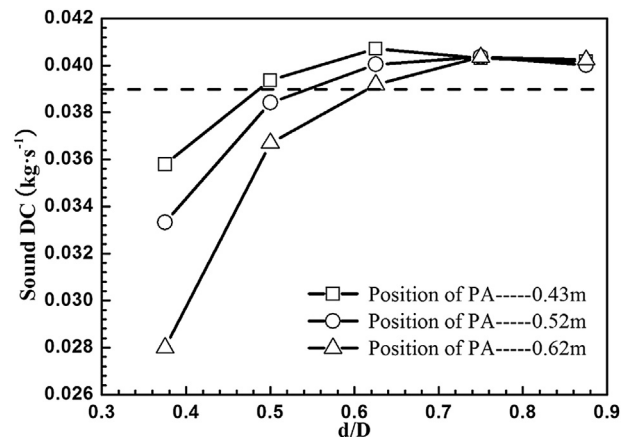


Fig. 8. The variation of sound DC as a function of the ratio d/D at different PA positions.

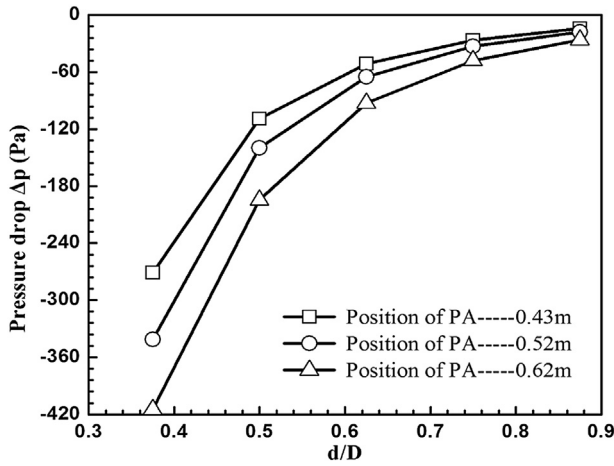


Fig. 9. The variation of pressure drop with the ratio d/D .

PA. The efficiency of the engine with PA increases compared to the case without PA when the $d/D \geq 0.5$. Especially, the maximal efficiency increases by 10% with the $d/D = 0.75$, PA position of 0.43 m away from the ambient end of the regenerator.

4.6. Oscillatory flow field

Fig. 11 shows the diagram of the pressure wave while the engine levels off. Four points in one cycle are chosen to illustrate the effects at phases of θ_1 , θ_2 , θ_3 and θ_4 . The streamlines at the position of PA, the elbow and the resonator junction are illustrated in Fig. 12. When the alternating current flows through the cross-sectional area with a sudden change and turns around, vortices with different intensity and sizes form. At the elbow, the periodic separation and converge of the alternating current because of turning around control the formation of vortices. The sudden variation of velocity due to abrupt change of the cross-sectional area intensifies the formation of vortices. The number and intensity of vortices decrease when the working fluid flows anticlockwise. The intensity and size of vortices around PA vary with time because the alternating flow experiences the sudden expansion and shrink of the cross-sectional area back and forth. The fluid separates and converges periodically at the resonator junction, thus generating vortices. It should be noted that PA aggravates the formation of vortices by narrowing the cross-sectional area. The formation and

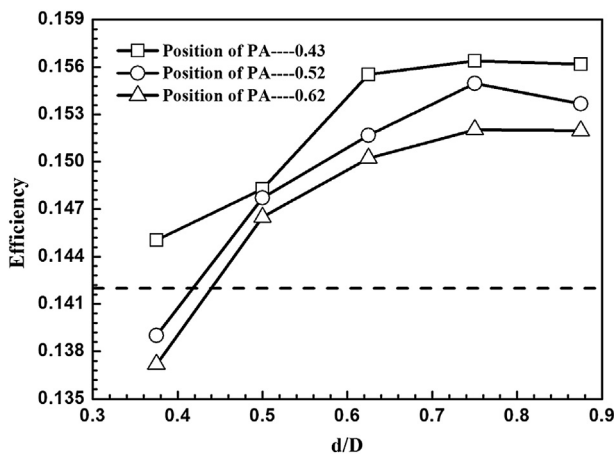


Fig. 10. The variation of efficiency as the function of the ratio d/D .

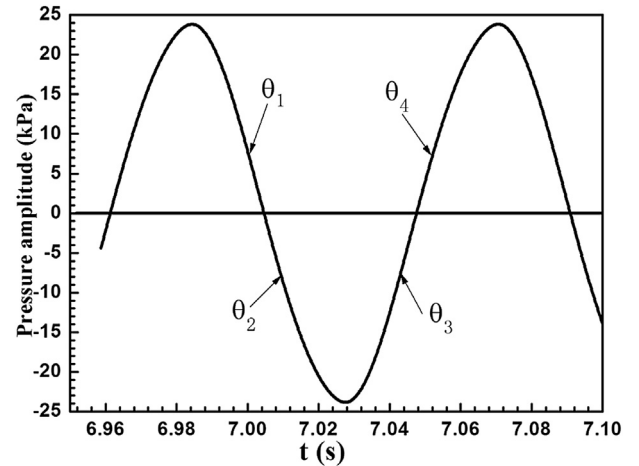


Fig. 11. Pressure wave when the oscillation stabilizes, θ_1 , θ_2 , θ_3 , θ_4 are chosen representing different phases in one cycle.

shedding of vortices could affect the distribution of velocity and temperature, causing unnecessary energy losses [25,26]. Therefore, it is helpful to optimize the structure of PA by analyzing the cause and position of vortexes formation.

Based on the previous analysis, the acoustic power decreases with smaller inner diameter of PA while increases with larger inner diameter of PA. The sound DC is smaller with smaller inner diameter of PA while is larger with larger inner diameter of PA. The reason that causes the variation of acoustic power and sound DC will be analyzed in the following section.

5. Discussions

5.1. Influence analysis of PA

The acoustic impedance at the ambient end of the regenerator determines the acoustic power flow into the regenerator. And the acoustic power flow into the regenerator can be approximately expressed as [6]

$$W_c = \frac{1}{2} \text{Re} [p_1 \tilde{U}_1] = \frac{|p_1|^2}{2 \text{Re}[Z]} \quad (19)$$

where $Z = p_1/U_1$ is the acoustic impedance at the ambient end of the regenerator, p_1 and U_1 are pressure amplitude and volume flow rate amplitude. For given pressure amplitude at the ambient end of the regenerator, high acoustic impedance will reduce acoustic power flow into the regenerator. PA can accelerate velocity by narrowing the cross-section area, thus affecting the volume flow rate. The variation of volume flow rate at the ambient end of the regenerator as inner diameter and position of PA is shown in Fig. 13.

It can be seen that volume flow rate firstly increases and then decreases with the increase in inner diameter of PA. When the inner diameter of PA is fixed, volume flow rate decreases with the increase in position of PA. According to Eq. (19), the increase of volume flow rate decreases acoustic impedance at the ambient end of the regenerator, and then decreases the acoustic impedance. As such acoustic power flow into the regenerator is increased. When the inner diameter is smaller ($d/D = 0.375$), the volume flow rate decreases compared to the case without PA, thus acoustic power flow into the regenerator is reduced. PA with the larger inner diameter ($d/D \geq 0.6$) increases the acoustic power flow into the regenerator.

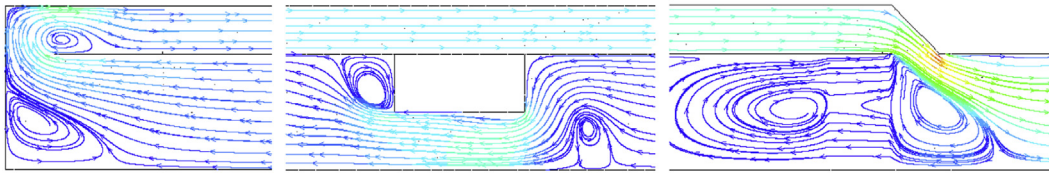
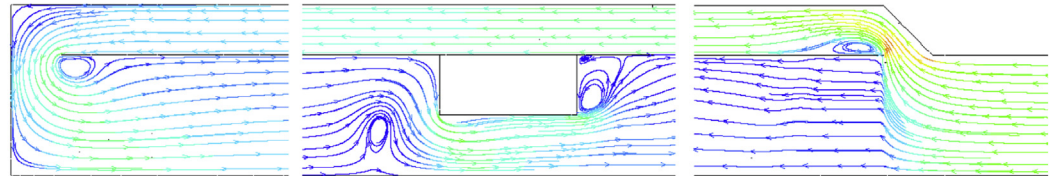
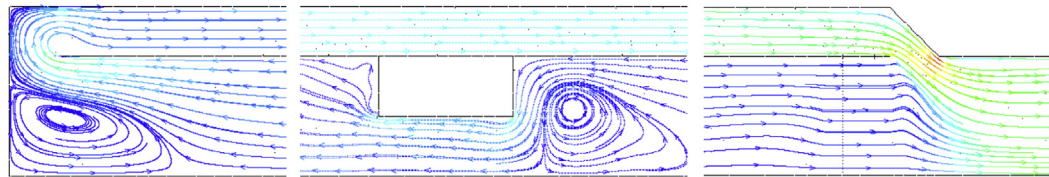
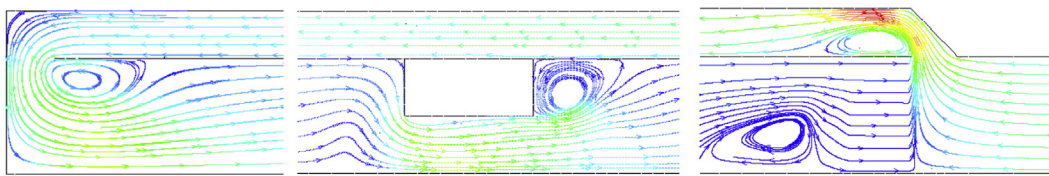
θ_1 : θ_2 : θ_3 : θ_4 :

Fig. 12. Streamline at different moments in one cycle for three different positions: the elbow (left), PA (middle), and the resonator junction (right).

However, when the alternating current flows through PA, it will make a transition from a large area to a small one, which can produce additional pressure drop and power dissipation called “minor losses”. The minor losses can be evaluated by a resistance, which is given by [14]

$$R_{PA} = \frac{4\rho_m u}{3\pi a_s} (K_B + K_S). \quad (20)$$

where R_{PA} denotes the acoustic resistance of PA, as represents the cross-sectional area of PA, ρ_m and u is the mean density and velocity of working gas at the position of PA, K_B and K_S are local resistance loss coefficients. Fig. 14 shows the variation of the resistance with the position and inner diameter of PA.

It can be seen that the resistance decreases with the increase in inner diameter of PA, and decreases with the position of PA. However, the impact of the position of PA weakens as inner diameter of PA increases. The power dissipation increases with R_{PA} . Therefore, the change of the resistance replacing PA reflects the variation of power dissipated by PA. It is not surprising that the smaller resistance means less power dissipated by PA, which has less effect on the engine.

Overall, the acoustic power is determined by the two combined effects above. On one hand, PA changes the acoustic impedance at the ambient end of the regenerator by narrowing the cross-sectional area. On the other hand, PA also dissipates power which can be measured by PA's resistance. When the inner diameter of PA is smaller ($d/D = 0.375$), the decrease of volume flow rate results in the decrease of acoustic power flow into the regenerator, and at the same time power dissipated by PA is maximal. The two effects both

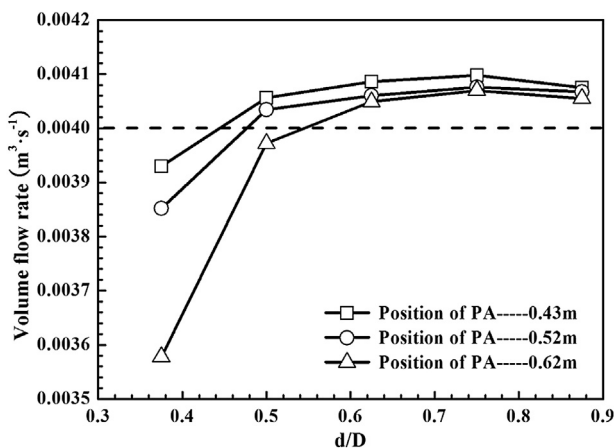


Fig. 13. The variation of volume flow rate at the ambient end of the regenerator with the ratio d/D .

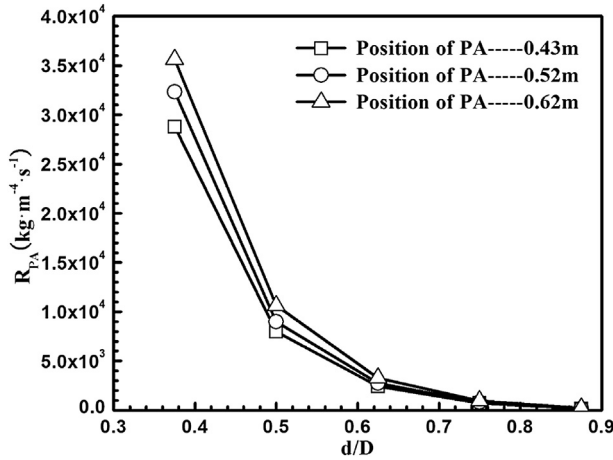


Fig. 14. The variation of resistance of PA with the ratio d/D .

worsen the performance and result in the obvious decrease of acoustic power. When inner diameter of PA is larger, the volume flow rate increases and power dissipated by PA decreases, which implies the more acoustic power flow in the regenerator gets the run upon the power dissipated by PA. Therefore, the performance is promoted. In addition, the volume flow rate increases and R_{PA} decreases with the decrease of PA's position. The closer position away from the hot end of the regenerator is more helpful to improve the performance. In summary, the larger inner diameter and closer position of PA is favorable for the system.

5.2. Analysis of sound DC

Sound DC in the loop inevitably causes unnecessary heat losses, lowering the efficiency of heat-to-sound energy conversion. To suppress sound DC is an important way to improve the performance of thermo-acoustic Stirling engine.

At the cold end of the regenerator, we can rewrite the first term on the right hand of Eq. (18) following Gedeon [27]

$$\text{Re}[\rho_1 \tilde{U}_1] / 2 = \rho_m W_c / p_m. \quad (21)$$

So sound DC can be rewritten

$$\dot{M}_2 = \rho_m W_c / p_m + \rho_m U_{20}. \quad (22)$$

According to Eq. (22), the variation of sound DC is closely related to acoustic power W_c and the second order volume flow rate U_{20} . On one hand, sound DC increases with acoustic power. On the other hand, the alternating current flowing through PA can generate pressure drop, producing U_{20} which can counteract a part of sound DC. When inner diameter of PA is smaller, acoustic power is minimal and pressure drop is maximal, leading to remarkably small sound DC. Sound DC with the ration $d/D = 0.5$ is determined by relative change of W_c and U_{20} . If the increase of sound DC caused by W_c is smaller than the reduction of sound DC offset by U_{20} , sound DC would decrease. Sound DC with PA of the ratio $d/D = 0.625$, 0.75 and 0.875 increases mainly because of the increase in W_c . Under these conditions, pressure drop produced by PA is so small that U_{20} cannot offset the increase in sound DC.

6. Conclusions

PA creates a strong flow condition in which particle velocity is accelerated by narrowing the cross-sectional area. On one hand, the

acoustic impedance at the ambient end of the regenerator is changed, which determines the acoustic power flow into the regenerator. On the other hand, PA also dissipates part power because of the sudden change of the cross-sectional area. The two combined factors both determine the effect of PA on the performance. In this study, the impact of these factors was investigated systematically by changing inner diameter and position of PA. The conclusions can be drawn as follows.

- (1) The variation of resonant frequency with PA is small despite how the position or inner diameter of PA changes. The reason is that the resonant frequency is mainly determined by the resonator, and the structural change after using PA is so small that it does not arouse the larger change of the resonant frequency.
- (2) Pressure amplitude and acoustic power are higher with the larger inner diameter of PA and the nearer distance from the hot end of the regenerator. Larger inner diameter and closer position of PA decrease the acoustic impedance at the cold of the regenerator, which increases the acoustic power flow into the regenerator. Concurrently, PA with larger diameter and closer position dissipates less power. Thus, the performance is promoted. The maximum values of pressure amplitude, acoustic power and efficiency increase by 3%, 4.5% and 10%, respectively.
- (3) Sound DC in the loop decreases with inner diameter of PA, but pressure drop generated by PA increases with inner diameter of PA. The bigger pressure drop can suppress the sound DC more effectively. When the inner diameter of PA is 30 mm, acoustic DC decreases by 28%.

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Nomenclature

Symbols

C_2	inertial resistance coefficient
d	diameter of PA, mm
D	diameter of tube, mm
E	internal energy, W
h	enthalpy, W
R	resistance, $\text{kg m}^{-4} \text{s}^{-1}$
k	thermal conductivity coefficient, $\text{W m}^{-1} \text{K}^{-1}$
L	length, m
a_s	cross-sectional area of PA, m^2
K	local resistance loss coefficient
p	pressure, Pa
r	hydraulic radius, mm
S_i	source, kg s^{-2}
T	temperature, K
u	particle velocity, m s^{-1}
U	volume flow rate, $\text{m}^3 \text{s}^{-1}$
x	axis position, m
Δp	pressure drop, Pa
a	sound velocity, m s^{-1}
W	acoustic power, W

Greek symbols

β	penetration coefficient
μ	coefficient of kinetic viscosity, $\text{kg m}^{-1} \text{s}^{-1}$
τ	stress, Pa
ω	angular frequency, s^{-1}
v	velocity, m s^{-1}
ρ	density, kg m^{-3}
φ	phase angle, $^{\circ}$

Superscripts

$\rightarrow$	vector
$\sim$	complex conjugation

Subscripts

c	cold end of the regenerator
f	working fluid
x	axial direction
res	resonator
B	expansion
m	mean
h	hot end of the regenerator
s	solid
r	radial direction
reg	regenerator
S	contraction

Other

$\parallel$	fluctuation amplitude
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